Non-DETERMINISTIC SCALES USED in TRADITIONAL TURKISH MUSIC

Can Akkoç, Ph.D.

University of South Alabama, College of Medicine, Mobile, AL, USA 36688
cakkoc@usouthal.edu / can193849@yahoo.com

Abstract

Non-deterministic pitch scales observed in traditional Turkish music are viewed as distributions along the pitch axis. This thesis contrasts with the conventional norm of using deterministic scales, where pitches are discrete fixed points on the number line (the pitch axis), as in fretted and keyboard instruments. In this study actual frequencies used by master musicians during an improvisation are measured in Hertz and analyzed with the long term goal of characterizing the underlying modal scales in the form of distributions. The potential of the measurement system, together with a variety of plausible mathematical analysis schemes is demonstrated on improvisations made by two prominent master musicians in a specific fundamental mode.

The Problem

The global problem is to construct mathematical characterizations for the modal scales used in traditional Turkish music. Such formulations are to be based entirely on acoustic measurements made on actual performances by indisputable masters.

Since a significant number of measured frequencies fall outside the nominal scale steps suggested by existing models, it is only natural to think of scales as distributions over the pitch axis. The said measurements are invariably going to lead to statistical distributions spread over specific intervals along the pitch axis, possibly presenting a unique pattern for each mode. Once these distributions are determined in raw form through measurements, mathematical analysis will be in order to establish their shape and positioning. This paper is an attempt to initiate research and stimulate debate on the mathematical foundations of scales made up of pitch distributions as used in traditional Turkish music. Furthermore, the concept of musical scales in the form of distributions may have universal applications, lending itself to modeling other musics of the world.

The Concept of Taksim and Maq̃am

An improvisation, or *taksim*[1] as it would be called in musical circles, can be viewed as a journey within the musical sound structure of the underlying mode[2], subject to a set of rules, some of which are loose and some very strict. A taksim is a musical creation that displays the generic characteristics of the underlying maq̃am. During a taksim the listener is *carried* in soul into a spiritual world emulated by the maq̃am. The performer is not bound by the rigid rules of standard musical forms of traditional Turkish music. However, the taksim has to adhere to a certain standard of performance and technique dictated by the style, era, instrumentation, etc. Also it is not unusual to find certain 'stock tunes' being used by different performers in a given maq̃am.

The underlying maq̃am can be viewed as the musical *universe or habitat* where the taksim takes place. Every maq̃am is made up of an underlying scale, given a priori as a sequence of pitch distributions with its unique statistical architecture. During the course of a taksim the performer draws pitches from the said distributions, going up and down the scale, following a non-deterministic road map admissible for the maq̃am. The issue of melodic progression, or

[1] *Taksim* is a term of Arabic origin which has been incorporated into Turkish to signify a specific form of improvisation.

[2] The term mode is used here in place of the Arabic word 'maq̃am'. In this author's judgment mode is a poor substitute for the original term, in the sense that it does not carry all the connotations and ramifications of the word maq̃am. See Powers (1980) for a comprehensive monograph on the concept of maq̃am. From here on the term maq̃am is going to be preferred over the word mode. There appears to be around 120+ different maq̃ams in traditional Turkish music.

more generally the problem of determining the set of all admissible melodic progressions (road map) for a given maq̃am is known as the *seyir*[3] problem. Every maq̃am has its own unique set of uncountably infinite admissible paths. The musician can not stray beyond the seyir allowed for the maq̃am at hand.

In view of these preliminary considerations it becomes clear that any scientific investigation into the mathematical structures of traditional Turkish music needs to address the issues of (a) determining the underlying scales for different maq̃amāt[4], (b) characterizing the set of all admissible paths for each of the maq̃amāt, making up the grand structure of this music.

In this article we focus only on problem (a) and propose a general mathematical scheme for establishing modal scales in the form of pitch distributions.

Determination of Pitch on Actual Performances

Direct measurements were made on actual performances (taksim) of solo instruments, taken from two indisputable masters of traditional Turkish music. The fundamental frequency was determined in Hertz every milli-second throughout the entire taksim within a resolution of 3 cents. The measurement system consisted of a 14-bit pitch-to-MIDI sound card with pitch-bend[5] capability installed in a personal computer. The card was sampling at the rate of 22,000 samples per second. Improvisations previously recorded on audio tapes using a dynamic microphone were fed into the sound card, to be encoded in midi format in real time. The output in standard midi format was processed in non-real time to access (a) the fundamental frequency and changes in fundamental frequency for every 'midi event' as a function of time (milliseconds), (b) total duration of every frequency used throughout the taksim. Information obtained in (a) represents the sequential ordering of sounds, while (b) leads to a histogram

[3] *Seyir* is of Arabic origin (sayr) with the generic meaning 'path'. In the context of music theory it is used in present day Turkish to mean the path a melody follows. For a more precise and abstract definition we are going to use the term seyir to mean the *sequential order of sounds* in a musical piece.

[4] Maq̃amāt is the plural form of the term maq̃am.

[5] Pitch-bend is a term used in the MIDI protocol to indicate sounds (pitches) that do not exist in the diatonic scale in western music. Such sounds are reached by 'bending' the anchor sounds in the 12-tET in either direction.

displaying the *frequency of occurrence* of all fundamental frequencies used during the course of the taksim.

Graphical Displays: TIME-PLOTS

Data extracted from midi files is displayed graphically in two different forms: *(1) Time plots (time vs frequency), (2) Histograms (frequency vs duration/frequency of occurrence)*. Time is measured in seconds and displayed in 'percent' of the total length of the entire taksim. Frequency is measured in Hertz and displayed in 'cents' measured from the tonic[6]. Results reported in this section demonstrate the road map followed by the master musician during a taksim in a specific maq̃am. These measurements will serve as the database for the entire research project.

Figures (1,2) are two snapshots taken from a taksim made by master musician neyzen Niyazi SAYIN on the *mansur ney* [7] and in the Uşşak mode. The entire taksim is partitioned into 14 frames, and through a complete set of such pictures one can literally *walk* through the taksim in non-real time, observing in detail what the performer is *doing* every milli-second.

Fig.1 is a sample time plot, the first 15 percent, from the said taksim. Niyazi SAYIN enters the taksim on the note named *Dügâh*[8]

[6]Suppose f_1 and f_2 are two pitches, measured in Hertz, and c_1 and c_2 their respective deviations, measured in cents, from a common reference pitch R. Then the algebraic relations between these measures are given by $f_1 = R * 2^{\frac{c_1}{1200}}$ and $f_2 = R * 2^{\frac{c_2}{1200}}$. Taking natural logarithms, $LN(f_1) = LN(R) + \frac{c_1}{1200} * LN(2)$ and $LN(f_2) = LN(R) + \frac{c_2}{1200} * LN(2)$, and subtracting the two equalities we get $LN(f_1) - LN(f_2) = \frac{LN(2)}{1200} * (c_1 - c_2)$, from which follows the expression $c_1 - c_2 = 1200 * [LN(f_1) - LN(f_2)]/LN(2)$ for the deviation, measured in cents, between the two pitches f_1 and f_2.

[7]'Ney' is the main wind instrument in traditional Turkish music. It is a cane (literally) open at both ends. Sounds are produced by blowing across one end, and it is sometimes referred to as 'end-blown cane'. It has finger holes to control the effective length of the vibrating air column. Sounds in between are produced by adjusting the tightness of the lips, positioning the head with respect to the axis of the cane, etc. Neys come in 12 different ranges and 'mansur' is the third lowest from the bottom. Master ney players are referred to as 'neyzen'.

[8]See Arel (1948) and Ezgi (1935) for a comprehensive description of names used for notes in Turkish music. Deterministic sound scales (scales made up of fixed pitches) currently used in Turkish music were proposed by Hüseyin Sadettin Arel and Suphi Ezgi around 1930. From here on they will be referred to as the 'Arel-Ezgi' scale. Note names are indicated in the figures (time plots) on the right hand side of the box next to the vertical axis. In addition, a table showing the frequencies, in Hertz, of notes used in Turkish music together with deviations, in cents, from the tonic for the Uşşak mode is provided at the end of this paper. The Arel-Ezgi scale is based on 53 tone equal temperament, with only a select subset of 24 unequally-spaced pitches used in each octave.

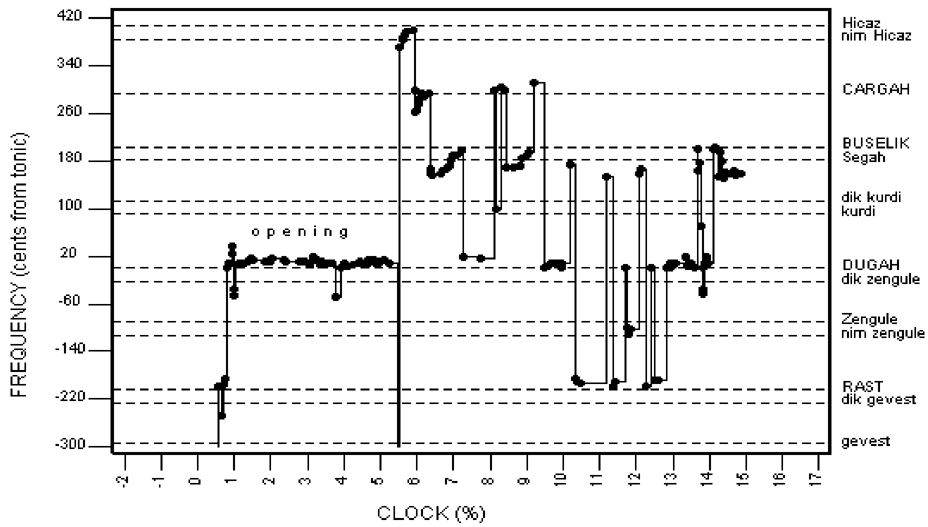

Figure 1: Time-Plot taken from a taksim in the Uşşak Mode, neyzen Niyazi SAYIN, Mansur Ney, 0-15%; horizontal axis: CLOCK (percent of the total time for the entire taksim) vs vertical axis: FREQUENCY (cents measured from the tonic).

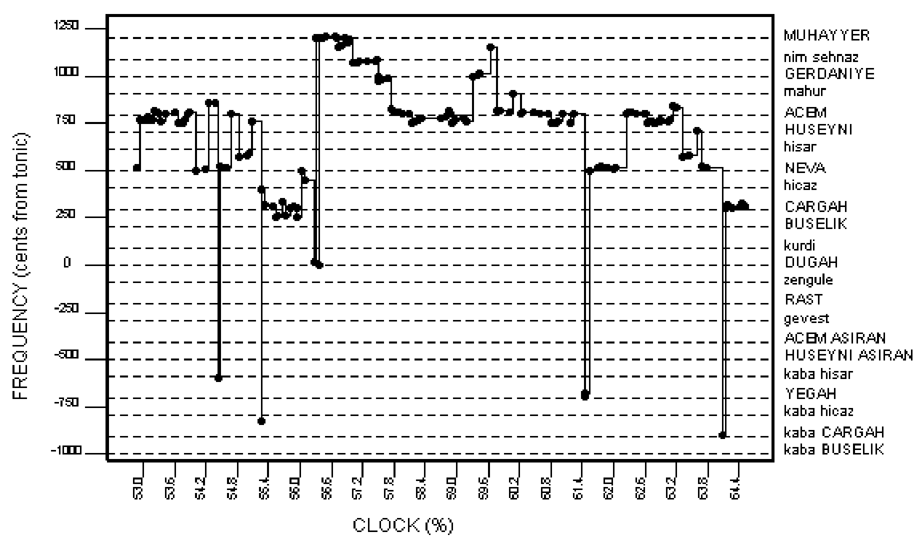

Figure 2: Time-Plot taken from a taksim in the Uşşak Mode, neyzen Niyazi SAYIN, Mansur Ney, 53-64%; horizontal axis: CLOCK (percent of the total time for the entire taksim) vs vertical axis: FREQUENCY (cents measured from the tonic).

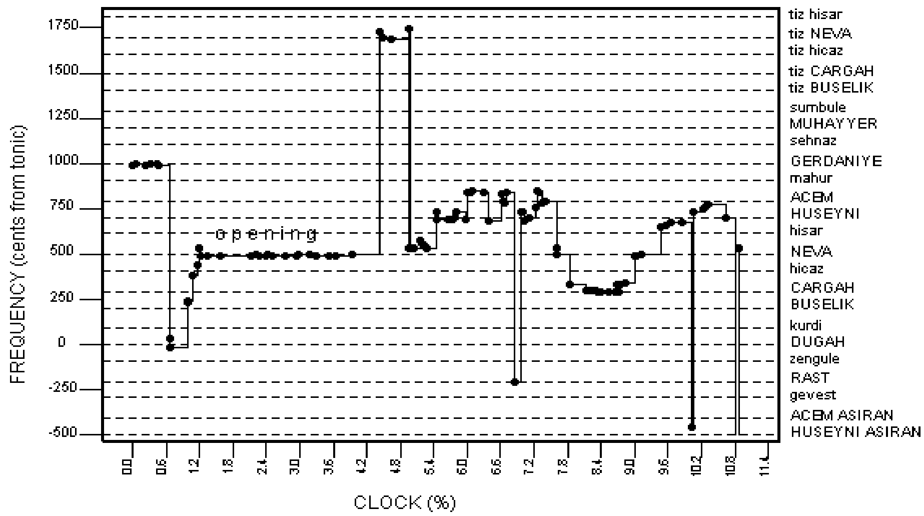

Figure 3: Time-Plot taken from a taksim in the Uşşak Mode, İhsan Özgen, kemençe; 0-11%; horizontal axis: CLOCK (percent of the total time for the entire taksim) vs vertical axis: FREQUENCY (cents measured from the tonic).

which is the tonic for the Uşşak mode[9], climbs all the way up to the Hicaz zone, modulates gradually down to Rast, back up to Dügâh, and this frame ends in the Buselik-Segâh zone.

Fig.2 is another sample from the same taksim covering the time interval 53-64%. The frame begins around Acem, modulates down to Çargâh, moves up to Muhayyer, the upper tonic, descends to Acem, and the frame ends at Çargâh.

Figures (3,4) are frames from a taksim by kemençeci (kemençe player) İhsan Özgen on the kemençe[10] in the Uşşak mode. Fig.3 is the first frame of a 17 frame time-plot. The performer enters the taksim on Nevâ and sits on it for a long time. Then he goes straight up to tiz Nevâ, the dominant note for the maq̃am, descends

[9]Different tunings are used for instruments with different registers. The table at the end of this paper shows nominal notes used in the Uşşak mode for Mansur and Bolahenk tunings. Mansur ney is played in the Mansur tuning. Bolahenk tuning is used for the kemençe, another instrument investigated in this paper.

[10]'Kemençe' is the main string instrument (bowed) in traditional Turkish music. It is not fretted and resembles a miniature pear-shaped cello. It is held in the lap in upright position and played with the bow facing away from the performer. Sounds are modulated by pressing the finger nails against the strings laterally, while the tip of the finger rests against the neck of the instrument.

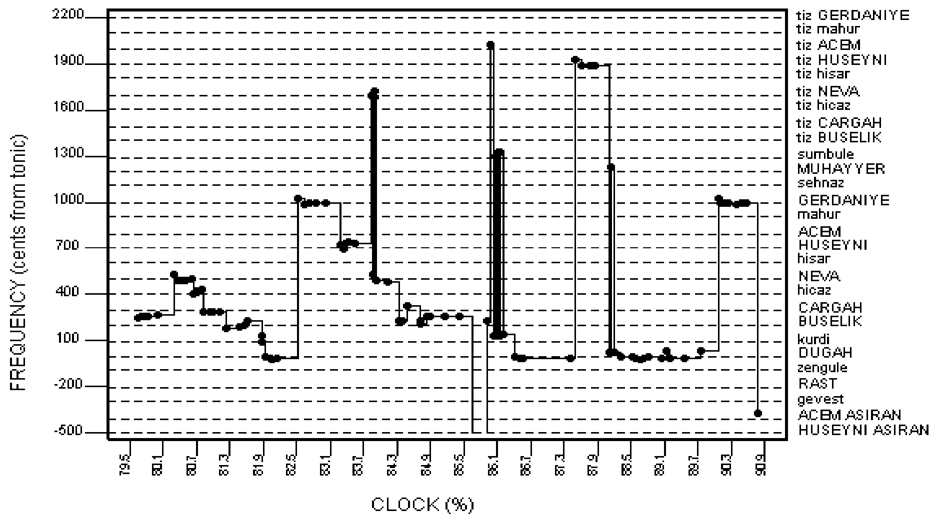

Figure 4: Time-Plot taken from a taksim in the Uşşak Mode, İhsan Özgen, kemençe; 71-82%; horizontal axis: CLOCK (percent of the total time for the entire taksim) vs vertical axis: FREQUENCY (cents measured from the tonic).

to Hüseyni, and down to Çargâh, climbing gradually to Acem, finally returning to Nevâ at the end of the frame.

Fig.4 is another frame close to the end of the same taksim. It opens around Çargâh, drops down to Dügâh, goes up to Gerdaniye and then to tiz Nevâ, down to Çargâh, a sudden climb all the way up to tiz Acem, drops all the way down to Dügâh, up to tiz Hüseyni, back down to Dügâh, and resting at Gerdaniye at the end of the frame.

Graphical Displays: HISTOGRAMS

The problem of constructing mathematical models for the modal scales was approached through a statistical concept called *cluster analysis*, discussed in the next section. A select sample of graphical results in the form of local histograms are presented here for demonstration purposes. It is conjectured that this statistical scheme may be useful in constructing complete characterizations of all modal scales in traditional Turkish music.

Figures (5,6) are local histograms of clusters from the Uşşak tak-

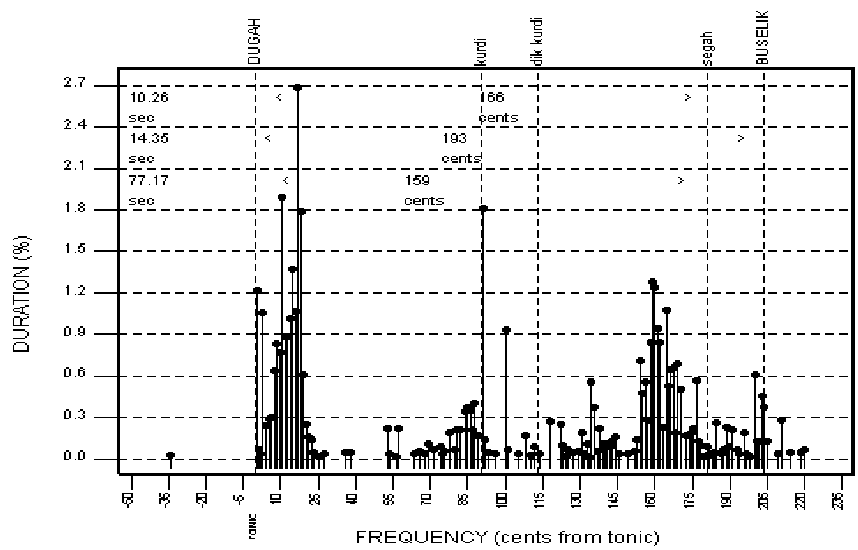

Figure 5: Histogram taken from a taksim in the Uşşak Mode, neyzen Niyazi SAYIN, Mansur Ney; interval 0-228 cents; horizontal axis: FREQUENCY (cents measured from the tonic) vs vertical axis: DURATION/cumulative (percent of the total time for the entire taksim).

sim by neyzen Niyazi SAYIN. The height of each vertical 'pole' represents the cumulative duration, in percent of the total, of each pitch (frequency) used throughout the entire taksim. In general a given pitch is visited several times during the course of the taksim at different points in time. A computer program scans all frequencies used in the taksim and detects the duration for each usage, adding the duration times in a cumulative manner. Names of nominal frequencies are indicated by dashed vertical lines on the histograms.

Fig.5 displays two clusters as determined by cluster analysis. The lower cluster (frequency-wise) is slightly above the tonic Dügâh per the Arel-Ezgi scale. Its mean ('medoid' from cluster analysis) is 14 cents higher than the nominal tonic. Its range is 25 cents for the *core*[11]. The weight[12] of its core is 24%. Its compact-

[11]Partitioning all frequencies used in the entire taksim into 11 clusters turned out to be one of several optimal solutions for the set. The cluster containing the concentration just to the right of Dügâh (440Hz) runs from 428Hz (-48cents) to 463Hz (88cents) for a range of 136 cents. However, there are empty zones in the said interval with no frequencies used. We are going to refer to such concentration zones close to the mean (medoid) as the 'core' of the cluster.

[12]Total duration of all sounds in a cluster is going to be referred as the 'weight' of that

ness ratio[13] is 24%/25cents=0.96%/cent. These four parameters `(mean,range,weight,compactness ratio)`, viewed as an *ordered quadruple*, are going to be used for characterizing the statistical properties of the clusters along the frequency axis.

The higher cluster in Fig.5 which runs from 464Hz (92cents) to 503Hz (232cents) has the characteristics (162cents,140cents,24%, 0.17%/cent). This cluster covers the notes dik Kurdi, Segâh, and Buselik. The cluster mean is 19 cents below Segâh. Theoretically there should be no sounds other than Segâh in this zone, per the Arel-Ezgi scale. The fact that this master musician is using the 140 cent interval around Segâh almost like a continuum or 'smear' suggests a fundamental shortcoming for the said theoretical model.

Looking at sequential data (time vs frequency) in the database pertaining to these two clusters, the following observations were made. At time station 10.26 seconds the melodic sequence goes from 442Hz in the cluster around Dügâh to 487Hz in the adjoining cluster Segâh. This sound interval at this particular moment in the melodic progression is 166 cents wide. At a later time, 14.35 seconds into the taksim, the performer re-visits these same two clusters in succession, selecting an interval 193 cents wide, while at 77.17 seconds the melodic sequence makes a jump 159 cents wide.

Fig.6 is a snapshot covering two adjacent clusters from the higher registers in the Uşşak mode. The core around Çargâh has the ordered quadruple (307cents,22cents,11%,0.49%/cent). The mean is 13 cents higher than the nominal value in the Arel-Ezgi scale. The higher cluster with its core close to Nevâ, the dominant note, is at (516cents,25cents,6%,0.24%/cent). The mean is 17 cents higher than the nominal scale.

Fig.7 is a sample histogram from İhsan Özgen's taksim on the kemençe in the Uşşak mode. Two clusters were selected from a total of 11 clusters for the whole taksim. The lower cluster has a core of (297cents,40cents,10%,0.25%/cent). The cluster itself is 117 cents wide (range) with an isolated spike resembling an outlier at 329 cents, weighing 2.8% by itself.

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cluster, measured in percent of the total time for the entire taksim.

[13] We define the 'compactness ratio' of a cluster by 'weight/range', measured in *percent/cent*. Compactness ratio can be viewed as a measure of spread, or how closely the sounds in the cluster are packed around its centroid. The degree of compactness of sounds in a given cluster varies directly with the compactness ratio, that is, the higher the compactness ratio, the more compact (closely packed) the sounds in the cluster. Loosely speaking compactness ratio might be considered to be close, in spirit, to what might be called a 'weighted' standard deviation.

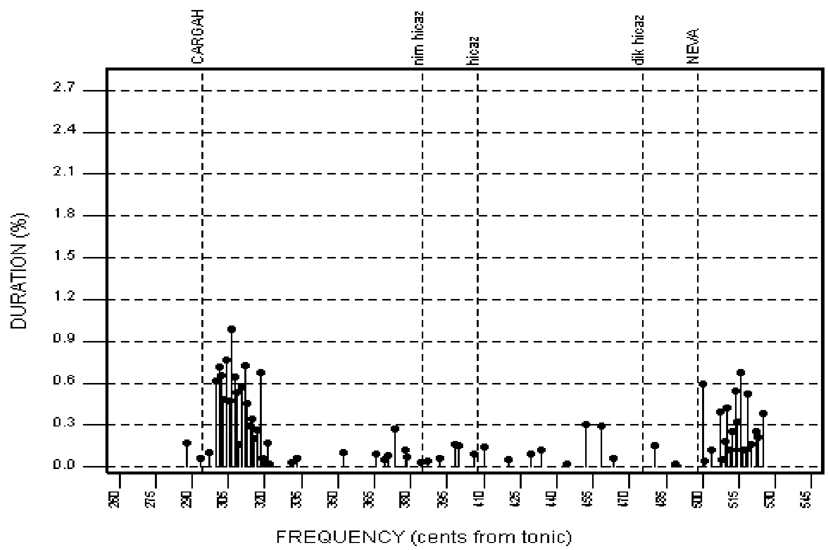

Figure 6: Histogram taken from a taksim in the Uşşak Mode, neyzen Niyazi SAYIN, Mansur Ney; interval 260-545 cents; horizontal axis: FREQUENCY (cents measured from the tonic) vs vertical axis: DURATION/cumulative (percent of the total time for the entire taksim).

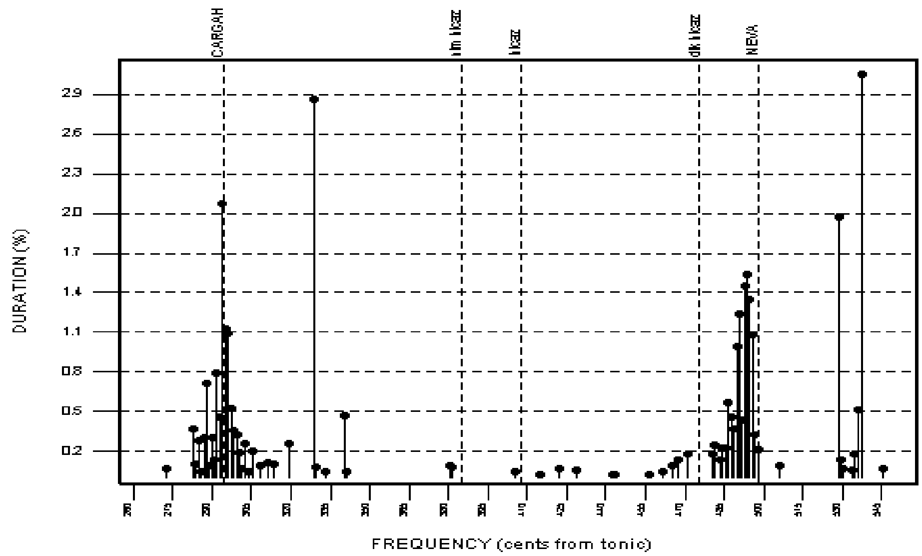

Figure 7: Histogram taken from a taksim in the Uşşak Mode, İhsan Özgen, kemençe; interval 260 to 545 cents; horizontal axis: FREQUENCY (cents measured from the tonic) vs vertical axis: DURATION/cumulative (percent of the total time for the entire taksim).

The higher cluster in Fig.7 is close to the dominant Nevâ and its core is of the form (496cents,17cents,11%, 0.64%/cent). This cluster also displays two high peaks outside the core at 529 cents (weight=1.9%), and Çargâh 538 cents (weight=3.0%).

Cluster Analysis

Cluster analysis is a statistical procedure[14] for partitioning a discrete data set into clusters based on the proximity of each element from the *medoid*, or centroid of the cluster. The user selects the number of clusters to be formed. The algorithm then places each element in the data set into one of the clusters based on its distance from a medoid as well as from neighboring clusters. A numerical measure of compactness is calculated for each cluster as well as an average for the entire set of clusters. This number can be viewed as a measure of compatibility of the data set being partitioned to the number of

[14]Please see Jain and Dubes (1988) and Kaufman and Rousseeuw (1990) for a comprehensive description of the procedure.

clusters selected by the user. By varying the number of clusters one can optimize the partitioning process. Optimal values for the number of clusters usually display local peaks at various locations.

Discussion

With the advent of accurate sound measuring technology, now it is possible to detect musical pitch in monophonic music at very small time intervals. Such a hardware-software system was used on two improvisations taken from two master musicians of traditional Turkish music. All sounds visited during the course of the taksim were detected, and a database was created to serve as a basis for mathematical analysis.

Any method of analysis that begins with an audio file rather than a score is of necessity an analysis of performance in the beginning. But in styles such as maq̃am music, where performers do not generally play from written scores, there is no way to get to the underlying musical scales except through the performance. It is recognized up front that untangling the many variables and interactions between musical form, the piece, and the individual performance is not an easy task. We believe this paper presents a first attempt to seriously deal with these issues in the context of traditional Turkish music.

On the improvisations analyzed in this study pitches (notes) used at different places in the taksim appeared to be clustering around a set of 'anchor' notes, displaying interesting characteristics of shape, size, and spread. The positioning of anchor notes appeared to be unique to the maq̃am the taksim was being created in, suggesting very strongly an underlying modal scale in the form of distributions. We propose here the concept of *modal scales in the form of distributions*, as opposed to scales made up of fixed pitches currently being used in Turkish music. Such distributions should be taken as sound clusters forming the raw database for a multiplicity of intricate pitch scales at the micro level. It is conjectured that as the taksim unfolds the musician is putting together impromptu micro intonations by drawing pitches from several clusters as building blocks. Different pitches may be drawn from a given cluster at different points in time to contribute to a micro scale depending upon the dynamic character of the multiplicity of intonations needed in the taksim. Observations reported on sound intervals created by the performer around the clusters at Dügâh and Segâh are examples that support this conjecture (Fig.5). Two consecutive clusters are visited back to back at different points in time, and at each visit the musician has selected different set of frequencies from the two clusters, thereby creating a variable micro scale over the said zone. The following question would be in order: during a melodic progression which factors determine pitch selection as the music visits different clusters in the underlying scale? The author offers the speculative model that the exact pitch to be played when the taksim 'gets' to a certain cluster is determined primarily by where the music has been, that is, which pitches have been used for the past several steps before coming to that cluster. This novel idea suggests the existence of a short term 'memory' as an integral feature of pitch sequences in traditional Turkish music.

There appears to be two different tunings operating at two different levels. At the macro level the underlying scales which are given a priori for the maq̃am in the form of distributions constitute a 'fixed' statistical scale. At the micro level temporary 'local' scales (time wise) kick in with an ever changing, dynamic structure. This micro tuning takes place and evolves within the *grand space* of the underlying distributions in the backdrop. Such delicate and sophisticated structures could very well form the skeleton of a multi-layer *vertical context*[15] within the one-dimensional manifold of monophonic music.

Sound clusters shown in the histograms exhibit statistical characteristics that may be of value in uncovering and interpreting the sound structures in Turkish music. In the case of mansur ney anchor notes (medoids) were positioned consistently higher (around 10-15 cents) than the nominal notes suggested by the Arel-Ezgi scale for the said maq̃am. Furthermore, the clusters around the tonic and the dominant note were more closely packed (higher compactness ratio) than notes of lesser importance. The first observation is difficult to explain. The ney might have been made to a slightly higher tuning than the theoretical model. The style of the musician; positioning of the head with respect to the cane, tightness of fingers on the holes, intensity of blowing (non-linear effects), the teachings

[15] The author finds different musics of the world evolving in different directions over the course of time. Every music seems to be in search of some form of structural 'enrichment' to drive its evolution. The author likes to think of this process as a quest for *multi-dimensionality*. Turkish music appears to have acquired structural 'richness' through intricate tuning patterns. The standard term 'vertical context' is used here in the spirit of multi-dimensionality.

of his master could be several points one could speculate about for this 'discrepancy'. The latter observation might be easier to explain. The current model for scales appears to be more consistent with actual performances by master musicians around the tonic, the dominant, and the upper tonic. However, it has been observed to be fuzzy, sometimes fuzzy to the extent of completely missing what appears to be almost a smear created by the performer in certain zones along the pitch axis, for other notes.

In the case of kemençe deviations from nominal notes appeared to be random. In various taksims that were studied by the same kemençe player clusters around anchor notes corresponding to open strings for certain maq̃ams were observed for compactness relative to other notes in the scale. The idea was to determine whether open strings were being used for pitch reference. The evidence found at this point in the research project was not sufficient to support such a conjecture with statistical significance.

Comparing the two instruments[16] used in this pilot study, clusters for kemençe appeared to be generally wider than those for the ney. Such difference could due to a variety of factors. The kemençe player has unlimited latitude in selecting pitches within the statistical bounds for the maq̃am, since the instrument is not fretted. The author is inclined to believe this particular kemençe player is following the music more freely. Theoretically the kemençe can produce any pitch within its range corresponding to points on the frequency axis (the number line) as a continuum. For the ney it is a totally different story. Playing the ney is a fuzzy process where, in addition to how 'tight' each hole is stopped during modulations, positioning of the player's head can lead to deviations in tuning in the order of a semitone[17].

Cluster analysis is considered to be only a first approximation to the global problem of characterizing the modal scales in the form of distributions. The algorithm is open to fine tuning and possibly

[16]The two instruments used in this investigation and described briefly in footnotes 7 and 10 allow the performer unlimited freedom in intonation. The musician can produce any pitch, like a continuum, within the register of the instrument. This feature is so fundamental for Turkish music that any constraints on the sound production mechanism would hinder the performance. As such, fretted and keyboard instruments are technically not favored in traditional Turkish music.

[17]Another study directed at investigating the 'flexibility' of the ney in producing pitches that deviate from anchor sounds obtained by fully open or fully closed finger holes would definitely be a worthwhile project.

significant modifications for closer and more meaningful characterizations of individual clusters. In the histograms discussed here it was found necessary to refer to a 'core' inside each cluster with heavy concentration of sounds. Cluster analysis was designed to place every single frequency into a certain cluster. This is why most clusters exhibit empty subintervals within their ranges. Clearly a hierarchical modification of this technique needs to be developed for statistically accommodating compact cores within wide clusters.

It should also be realized that the data used in this preliminary study came from several taksims performed by only two master musicians on a variety of maq̃ams. A comprehensive study using taksims and formal compositions from numerous resources would be needed for a conclusive analysis, possibly opening the way for a universal characterization of the non-deterministic scales in Turkish music as well as others musics.

Acknowledgments

The central ideas presented in this article were first conceived during visits by the author to masters classes conducted by the late Dr.Emin KILIÇ KALE of Gaziantep, Türkiye (Turkey). Timeless interaction between the master and the aspiring apprentice allowed the author his first glimpse of how traditional Turkish Music was learned by 'passing the gift' from one generation to the next in a *dergâh*[18] setting. The author is indebted to Dr.Emin KILIÇ KALE and his students, in particular Mr.YILMAZ KALE, for this extraordinary opportunity.

During the author's brief tenure at Worcester Polytechnic Institute (Worcester, Massachusetts, USA) Professor Robert L. Norton, a close colleague, had so much faith in what the author was trying to do, he not only took upon himself the arduous task of bringing the author from the 'slide-rule age' into the 'microprocessor age', but went so far as writing customized software for pitch determination. Professor Norton deserves my endless gratitude.

Spring Hill College (Mobile, Alabama, USA) gave a timely grant to the author for purchasing hardware and software in making measurements on improvisations. This most valuable gift was a turning

[18]'Dergah' is the Turkish word for the 'house of a sûfi order', where traditionally music has been the preferred habitat for meditation and communication in sûfism.

point for the project. It introduced the author into the world of MIDI (musical instrument digital interface).

Two young students, Nathan Lott and Ryan Culpepper at the Alabama School of Mathematics and Science (Mobile, Alabama, USA) finally made things happen by writing customized software for converting coded midi messages into comprehensible forms. These two gentlemen also deserve my utmost gratitude for their never-ending service executed with an ever-smiling face.

Professor Michael Windham, chairman, Department of Mathematics and Statistics at the University of South Alabama (Mobile, Alabama, USA) introduced the author to cluster analysis. Dr. Mia Hubert of the University of Antwerp, Belgium took the time to modify a cluster analysis software to meet the special needs of the author. These two wonderful people have contributed immensely. The author recognizes their invaluable assistance.

Dr. Albert Lilly, a colleague from the Alabama School of Mathematics and Science (Mobile, Alabama, USA) demonstrated infinite patience in getting the author acclimated into the world of high-tech equipment and software. He deserves my endless thanks and appreciation for his never ending efforts.

The author's long time friend, humble sûfi, musician, instrument maker and ebrû artist Mr. Feridun Özgören of Boston deserves my special 'şûkran' for his endless sûfi patience in educating the author on the subtleties of traditional Turkish Music. Without this most valuable resource the project would never have lifted off the ground.

Master musicians of traditional Turkish Music, neyzen Niyazi SAYIN and kemençeci İhsan Özgen were most generous in providing the author with their own interpretations of the fundamental maq̃ams in the form of taksims. These recordings have been the main resource for 'authentic data' in this study. These two artists, coming from the dergâh era, have graciously allowed the author to use their taksims as demonstrations in preparing this article. The author wishes to express his deep gratitude to neyzen Niyazi SAYIN and kemençeci İhsan Özgen for their generosity and enthusiastic support during the inception of this project.

Notes Used in Traditional Turkish Music

The following table contains the names of a select sample of notes used in Turkish music together with their nominal frequencies in Hertz, as well as their deviations from the tonic for the Uşşak mode. Only the Mansur and Bolahenk tunings are shown. Frequencies are taken from the Arel-Ezgi scale.

Hertz	MANSUR	Cents	BOLAHENK	Cents
146.67	Kaba Yegâh	-1901.89	Kaba Rast	-1403.77
184.99	Kaba H. Aşiran	-1698.11	Kaba Dügâh	-1200.00
195.57	Kaba Rast	-1403.77	Kaba Çargâh	-905.66
208.79	Kaba Zengüle	-1290.57	Kaba Hicaz	-792.45
220.00	Kaba Dügâh	-1200.00	Yegâh	-701.89
231.82	Kaba Kürdi	-1109.43	Nim Hisar	-611.32
234.87	Kaba Dik Kürdi	-1086.79	Hisar	-588.68
244.26	Kaba Segâh	-1018.87	Dik Hisar	-520.75
247.48	Kaba Buselik	-996.23	Hüseyni Aşiran	-498.11
260.77	Kaba Cargâh	-905.66	Acem Aşiran	-407.55
293.34	Yegâh	-701.89	Rast	-203.77
329.99	Hüseyni Aşiran	-498.11	Dügâh (tonic)	0.00
347.71	Acem Aşiran	-407.55	Kürdi	90.57
371.21	Geveşt	-294.34	Buselik	203.77
391.14	Rast	-203.14	Cargâh	294.34
440.00	Dügâh (tonic)	0.00	Nevâ	498.11
586.69	Nevâ	498.11	Gerdaniye	996.23
659.97	Hüseyni	701.89	Muhayyer	1200.00
732.77	Eviç	883.02	Tiz Segâh	1381.13
742.41	Mahûr	905.66	Tiz Buselik	1403.77
782.28	Gerdaniye	996.23	Tiz Çargâh	1494.34
824.30	Nim Şehnaz	1086.79	Tiz Nim Hicaz	1584.91
835.15	Şehnaz	1109.43	Tiz Hicaz	1607.55
868.57	Dik Şehnaz	1177.36	Tiz Dik Hicaz	1675.47
880.00	Muhayyer	1200.00	Tiz Nevâ	1698.11
989.92	Tiz Buselik	1403.77	Tiz Hüseyni	1901.89
1173.38	Tiz Nevâ	1698.11	Tiz Gerdaniye	2196.23
1319.95	Tiz Hüseyni	1901.89	Tiz Muhayyer	2400.00

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